

問1 クラメルの公式を使って次の連立方程式を解け。

(1)

$$\begin{cases} 2x_1 + 4x_2 + 3x_3 = 11 \\ x_1 + x_3 = 3 \\ x_1 + 2x_2 = 4 \end{cases}$$

連立方程式を行列で表すと次のようになる。

$$\begin{pmatrix} 2 & 4 & 3 \\ 1 & 0 & 1 \\ 1 & 2 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 11 \\ 3 \\ 4 \end{pmatrix}$$

$$\begin{aligned} x_1 &= \frac{\begin{vmatrix} 11 & 4 & 3 \\ 3 & 0 & 1 \\ 4 & 2 & 0 \end{vmatrix}}{\begin{vmatrix} 2 & 4 & 3 \\ 1 & 0 & 1 \\ 1 & 2 & 0 \end{vmatrix}} = \frac{\{(11 \times 0 \times 0) + (3 \times 2 \times 3) + (4 \times 4 \times 1)\} - \{(4 \times 0 \times 3) + (3 \times 4 \times 0) + (11 \times 2 \times 1)\}}{\{(2 \times 0 \times 0) + (1 \times 2 \times 3) + (1 \times 4 \times 1)\} - \{(1 \times 0 \times 3) + (1 \times 4 \times 0) + (2 \times 2 \times 1)\}} \\ &= \frac{(0 + 18 + 16) - (0 + 0 + 22)}{(0 + 6 + 4) - (0 + 0 + 4)} = \frac{34 - 22}{10 - 4} = \frac{12}{6} = 2 \end{aligned}$$

$$\begin{aligned} x_2 &= \frac{\begin{vmatrix} 2 & 11 & 3 \\ 1 & 3 & 1 \\ 1 & 4 & 0 \end{vmatrix}}{\begin{vmatrix} 2 & 4 & 3 \\ 1 & 0 & 1 \\ 1 & 2 & 0 \end{vmatrix}} = \frac{\{(2 \times 3 \times 0) + (1 \times 4 \times 3) + (1 \times 11 \times 1)\} - \{(1 \times 3 \times 3) + (1 \times 11 \times 0) + (2 \times 4 \times 1)\}}{\{(2 \times 0 \times 0) + (1 \times 2 \times 3) + (1 \times 4 \times 1)\} - \{(1 \times 0 \times 3) + (1 \times 4 \times 0) + (2 \times 2 \times 1)\}} \\ &= \frac{(0 + 12 + 11) - (9 + 0 + 8)}{(0 + 6 + 4) - (0 + 0 + 4)} = \frac{23 - 17}{10 - 4} = \frac{6}{6} = 1 \end{aligned}$$

$$\begin{aligned} x_3 &= \frac{\begin{vmatrix} 2 & 4 & 11 \\ 1 & 0 & 3 \\ 1 & 2 & 4 \end{vmatrix}}{\begin{vmatrix} 2 & 4 & 3 \\ 1 & 0 & 1 \\ 1 & 2 & 0 \end{vmatrix}} = \frac{\{(2 \times 0 \times 4) + (1 \times 2 \times 11) + (1 \times 4 \times 3)\} - \{(1 \times 0 \times 11) + (1 \times 4 \times 4) + (2 \times 2 \times 3)\}}{\{(2 \times 0 \times 0) + (1 \times 2 \times 3) + (1 \times 4 \times 1)\} - \{(1 \times 0 \times 3) + (1 \times 4 \times 0) + (2 \times 2 \times 1)\}} \\ &= \frac{(0 + 22 + 12) - (0 + 16 + 12)}{(0 + 6 + 4) - (0 + 0 + 4)} = \frac{34 - 28}{10 - 4} = \frac{6}{6} = 1 \end{aligned}$$

$$\begin{cases} x_1 = 2 \\ x_2 = 1 \\ x_3 = 1 \end{cases}$$

(2)

$$\begin{cases} 2x_1 + 2x_2 + x_3 = 7 \\ x_1 + x_2 = 3 \\ 3x_2 + x_3 = 7 \end{cases}$$

連立方程式を行列で表すと次のようになる。

$$\begin{pmatrix} 2 & 2 & 1 \\ 1 & 1 & 0 \\ 0 & 3 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 7 \\ 3 \\ 7 \end{pmatrix}$$

$$\begin{aligned} x_1 &= \frac{\begin{vmatrix} 7 & 2 & 1 \\ 3 & 1 & 0 \\ 7 & 3 & 1 \end{vmatrix}}{\begin{vmatrix} 2 & 2 & 1 \\ 1 & 1 & 0 \\ 0 & 3 & 1 \end{vmatrix}} = \frac{\{(7 \times 1 \times 1) + (3 \times 3 \times 1) + (7 \times 2 \times 0)\} - \{(7 \times 1 \times 1) + (3 \times 2 \times 1) + (7 \times 3 \times 0)\}}{\{(2 \times 1 \times 1) + (1 \times 3 \times 1) + (0 \times 2 \times 0)\} - \{(0 \times 1 \times 1) + (1 \times 2 \times 1) + (2 \times 3 \times 0)\}} \\ &= \frac{(7+9+0) - (7+6+0)}{(2+3+0) - (0+2+0)} = \frac{16-13}{5-2} = \frac{3}{3} = 1 \end{aligned}$$

$$\begin{aligned} x_2 &= \frac{\begin{vmatrix} 2 & 7 & 1 \\ 1 & 3 & 0 \\ 0 & 7 & 1 \end{vmatrix}}{\begin{vmatrix} 2 & 2 & 1 \\ 1 & 1 & 0 \\ 0 & 3 & 1 \end{vmatrix}} = \frac{\{(2 \times 3 \times 1) + (1 \times 7 \times 1) + (0 \times 7 \times 0)\} - \{(0 \times 3 \times 1) + (1 \times 7 \times 1) + (2 \times 7 \times 0)\}}{\{(2 \times 1 \times 1) + (1 \times 3 \times 1) + (0 \times 2 \times 0)\} - \{(0 \times 1 \times 1) + (1 \times 2 \times 1) + (2 \times 3 \times 0)\}} \\ &= \frac{(6+7+0) - (0+7+0)}{(2+3+0) - (0+2+0)} = \frac{13-7}{5-2} = \frac{6}{3} = 2 \end{aligned}$$

$$\begin{aligned} x_3 &= \frac{\begin{vmatrix} 2 & 2 & 7 \\ 1 & 1 & 3 \\ 0 & 3 & 7 \end{vmatrix}}{\begin{vmatrix} 2 & 2 & 1 \\ 1 & 1 & 0 \\ 0 & 3 & 1 \end{vmatrix}} = \frac{\{(2 \times 1 \times 7) + (1 \times 3 \times 7) + (0 \times 2 \times 3)\} - \{(0 \times 1 \times 7) + (1 \times 2 \times 7) + (2 \times 3 \times 3)\}}{\{(2 \times 1 \times 1) + (1 \times 3 \times 1) + (0 \times 2 \times 0)\} - \{(0 \times 1 \times 1) + (1 \times 2 \times 1) + (2 \times 3 \times 0)\}} \\ &= \frac{(14+21+0) - (0+14+18)}{(2+3+0) - (0+2+0)} = \frac{35-32}{5-2} = \frac{3}{3} = 1 \end{aligned}$$

$$\begin{cases} x_1 = 1 \\ x_2 = 2 \\ x_3 = 1 \end{cases}$$

(3)

$$\begin{cases} 2x_1 + x_2 = 4 \\ 2x_2 + x_3 = 7 \\ x_1 + x_3 = 4 \end{cases}$$

連立方程式を行列で表すと次のようになる。

$$\begin{pmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 1 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 4 \\ 7 \\ 4 \end{pmatrix}$$

$$\begin{aligned} x_1 &= \frac{\begin{vmatrix} 4 & 1 & 0 \\ 7 & 2 & 1 \\ 4 & 0 & 1 \end{vmatrix}}{\begin{vmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 1 & 0 & 1 \end{vmatrix}} = \frac{\{(4 \times 2 \times 1) + (7 \times 0 \times 0) + (4 \times 1 \times 1)\} - \{(4 \times 2 \times 0) + (7 \times 1 \times 1) + (4 \times 0 \times 1)\}}{\{(2 \times 2 \times 1) + (0 \times 0 \times 0) + (1 \times 1 \times 1)\} - \{(1 \times 2 \times 0) + (0 \times 1 \times 1) + (2 \times 0 \times 1)\}} \\ &= \frac{(8+0+4) - (0+7+0)}{(4+0+1) - (0+0+0)} = \frac{12-7}{5-0} = \frac{5}{5} = 1 \end{aligned}$$

$$\begin{aligned} x_2 &= \frac{\begin{vmatrix} 2 & 4 & 0 \\ 0 & 7 & 1 \\ 1 & 4 & 1 \end{vmatrix}}{\begin{vmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 1 & 0 & 1 \end{vmatrix}} = \frac{\{(2 \times 7 \times 1) + (0 \times 4 \times 0) + (1 \times 4 \times 1)\} - \{(1 \times 7 \times 0) + (0 \times 4 \times 1) + (2 \times 4 \times 1)\}}{\{(2 \times 2 \times 1) + (0 \times 0 \times 0) + (1 \times 1 \times 1)\} - \{(1 \times 2 \times 0) + (0 \times 1 \times 1) + (2 \times 0 \times 1)\}} \\ &= \frac{(14+0+4) - (0+0+8)}{(4+0+1) - (0+0+0)} = \frac{18-8}{5-0} = \frac{10}{5} = 2 \end{aligned}$$

$$\begin{aligned} x_3 &= \frac{\begin{vmatrix} 2 & 1 & 4 \\ 0 & 2 & 7 \\ 1 & 0 & 4 \end{vmatrix}}{\begin{vmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 1 & 0 & 1 \end{vmatrix}} = \frac{\{(2 \times 2 \times 4) + (0 \times 0 \times 4) + (1 \times 1 \times 7)\} - \{(1 \times 2 \times 4) + (0 \times 1 \times 4) + (2 \times 0 \times 7)\}}{\{(2 \times 2 \times 1) + (0 \times 0 \times 0) + (1 \times 1 \times 1)\} - \{(1 \times 2 \times 0) + (0 \times 1 \times 1) + (2 \times 0 \times 1)\}} \\ &= \frac{(16+0+7) - (8+0+0)}{(4+0+1) - (0+0+0)} = \frac{23-8}{5-0} = \frac{15}{5} = 3 \end{aligned}$$

$$\begin{cases} x_1 = 1 \\ x_2 = 2 \\ x_3 = 3 \end{cases}$$