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THE SLIDING MODE VARIABLE STRUCTURE CONTROL BASED ON COMPOSITE REACHING LAW OF ACTIVE MAGNETIC BEARING

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Abstract. *The discrete time variable structure controller (VSC) based on combination reaching law is designed on the basis of the mathematical model of active magnetic bearing (AMB). Simulations and experiments show that the VSC strategy is more robust against external disturbances and model perturbation than PID, which has a wide application prospect in AMB area.*
Keywords: Variable structure control, Composite Reaching law, AMB

1. **Introduction.** Active Magnetic Bearing (AMB) has many advantages, such as no mechanical contact, high speed and accuracy, low power consumption, long service life and no environmental pollution, which has been considered as a promising new type of bearing used in the high-tech areas as energy resource, communication, mechanical industry, rotor dynamics and medical equipment. Yet the potential of AMB has not been dug sufficiently and AMB itself also has not reached the level of replacing other bearings, besides the high manufacturing costs and insufficient designing theories, the reason locates that there are still many unresolved control problems.

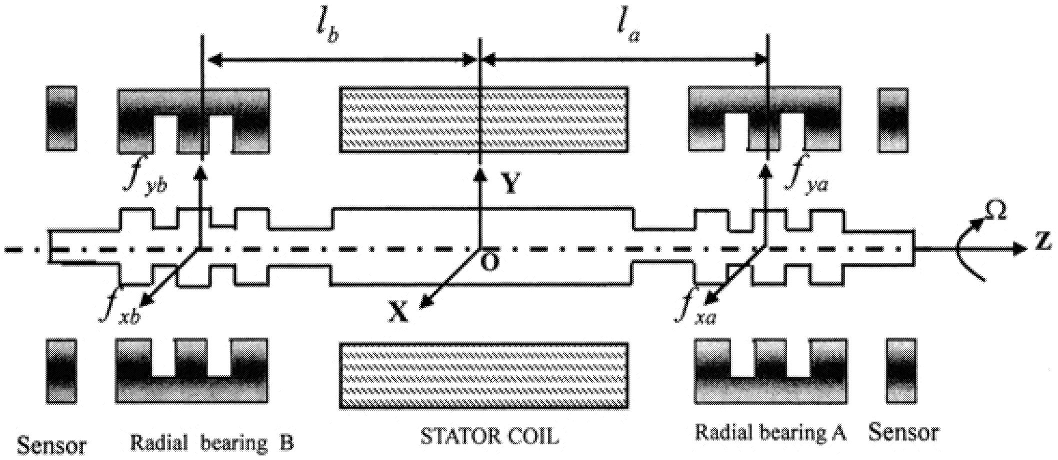

Figure 1. The structure map of AMB

The variable structure control has a special nonlinear characteristic. It has a wide application prospect in AMB area because its sliding mode represents invariance to the perturbation and outside perturbation motion under certain conditions.

2. **The Mathematical Model of AMB** [1]. Figure 1 shows the structure map of AMB. There are two pairs of differential transformer typed gap sensors and electromagnets on the left-right side and the up-down side of the axis.

Take x-axis direction for example; the electromagnetic force of the magnetic bearing of one degree of freedom is expressed as follows:

$$F = K_x x - K_i i = m\ddot{x} \tag{2-1}$$

where

$$\begin{cases} K_x = \dfrac{\partial F}{\partial x} = \mu_0 S N^2 \dfrac{I_0^2}{x_0^3} \\ K_i = \dfrac{\partial F}{\partial i} = \mu_0 S N^2 \dfrac{I_0}{x_0^2} \end{cases} \tag{2-2}$$

In the equation, μ_0 is the rate of the magnetism transmit in the air; N is the number of the circles of the stator coil; S is the area of the magnet pole; I_0 is the bias current; x_0 is the balance position of the stator. f_{xa}, f_{xb}, f_{ya}, f_{yb} are the rotor's electromagnetic force in the x, y direction of the A, B radial bearings. According to Newton's Second Law, the movement equations of the rotor's mass centre can be states as

$$\begin{cases} m\ddot{x}_0 = f_{xa} + f_{xb} \\ m\ddot{y}_0 = f_{ya} + f_{yb} \end{cases} \tag{2-3}$$

According to Inertia Law

$$\begin{cases} J_x \ddot{\theta}_x = -l_a f_{ya} + l_b f_{yb} - J_z \Omega \dot{\theta}_y \\ J_x \ddot{\theta}_y = l_a f_{xa} - l_b f_{xb} - J_z \Omega \dot{\theta}_x \end{cases} \tag{2-4}$$

In the equations, Ω is the angle speed of the rotor of z-axis. J_z is the inertia. $J_x = J_y$ are the inertia of x-axis and y-axis. θ_x, θ_y is the angle of the main bearing relative to x-axis and y-axis, x_a, y_a are the displacement that the rotor bias to the direction of x and y in the position of radial bearing A. x_b, y_b are the displacement that the rotator bias to the direction of x and y in the position of radial bearing B.

Combining all the equations above, we can get the state equations of the magnetic bearing system

$$\begin{cases} \dot{X} = AX + BU \\ Y = CX \end{cases} \tag{2-5}$$

$X = (x_a, x_b, y_a, y_b, \dot{x}_a, \dot{x}_b, \dot{y}_a, \dot{y}_b)^T$ is the state vector. $U = (i_{xa}, i_{xb}, i_{ya}, i_{yb})^T$is the control vector. i_{xa}, i_{xb}, i_{ya}, i_{yb} are the control currents in the bearing.

3. **The Design of Variable Structure Controller** [3,4]. The main axis close loop control system is shown as Figure 2, and the control algorithm is variable structure control. The controller is TMS320LF2407A DSP produced by TI Co, LTD which needed to discrete the math-module of the magnetic bearing. Take the a second-order system of one degree of freedom for example, after the discretization it becomes [2]

$$x(k+1) = Gx(k) + Hu(k) \tag{3-1}$$

where $G = e^{AT}$, $H = \int e^{AT} dt B$.

As to the digital control system shown in Figure 2, the variable structure control algorithm must be discrete as well. The handoff function is:

$$s(k) = C_e E = C_e(R - x(k)) \tag{3-2}$$

R is the given value of position, which is also the balance position of the magnetic bearing system. E is the deviation value, $x(k)$ is the math-module of the system after discretization.

For

$$R_1 = [r(k+1); dr(k+1)], \quad R = [r(k); dr(k)]$$

then

$$s(k+1) = C_e(R_1 - x(k+1)) = C_eR_1 - C_eGx(k) - C_eHu(k) \tag{3-3}$$

The rate of control is:

$$u(k) = (C_eH)^{-1}(C_eR_1 - C_eGx(k) - s(k+1)) \tag{3-4}$$

The Reaching Law is a typical control strategy of slide block variable structure control. By using this method, slide block campaign can be analyzed when system is on or around switching surface, and analyze the dynamic course of the limitation and weaken the influence of the shake to insure the system has a better character in the whole state space. This paper will use three different reaching rates, by analyzing their advantages and disadvantages to get a better strategy suitable for the control of magnetic bearing system.

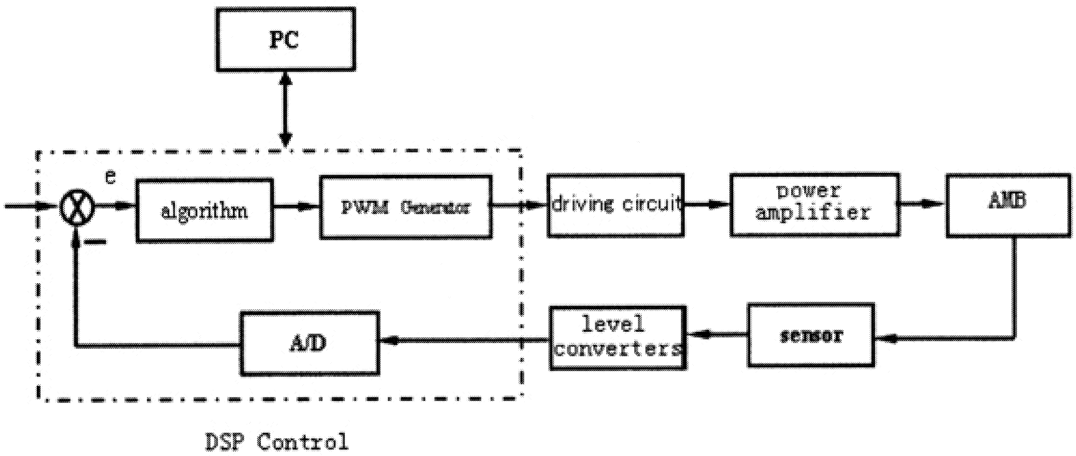

FIGURE 2. The digital control system

The discrete reaching law based on index reaching law is

$$s(k+1) = s(k) + T(-\varepsilon sgn(s(k)) - qs(k)) \tag{3-5}$$

We can get the discrete reaching rate based on variable speed reaching law

$$u(k) = (C_eH)^{-1}(C_eR_1 - C_eGx(k) - s(k) - ds(k)) \tag{3-6}$$

The switching area of index reaching law is a band no passing the original point with a εT width. In the stable state, slide block function cut between these two values, so a big stable shaking may come out. According to variable speed law, cut band is composed of two beams which pass the original point, and it also contains $s = 0$. In the stable state, the system can stay at the original point, which can lower shaking in the stable state, which means it has better stable characteristic. But when system comes into switching area, due to $||x(k)||_1$ is much bigger, the method of variable speed law may produce stronger shaking.

As shown in Figure 3 and Figure 4, if combining the index reaching law and the variable speed law together, the disadvantages of these two reaching laws can be overcome, and

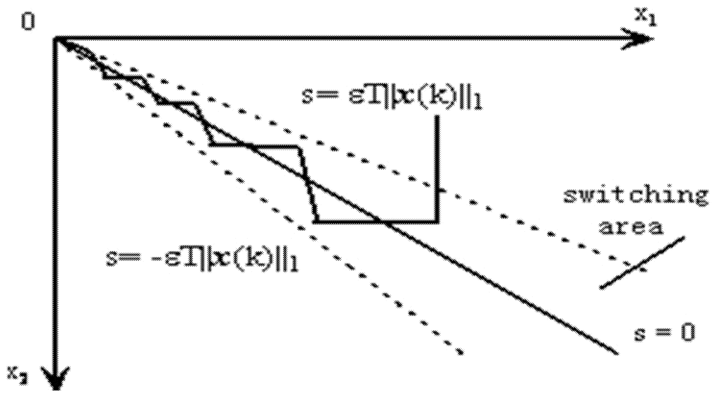

FIGURE 3. Switching area of index reaching law (I)

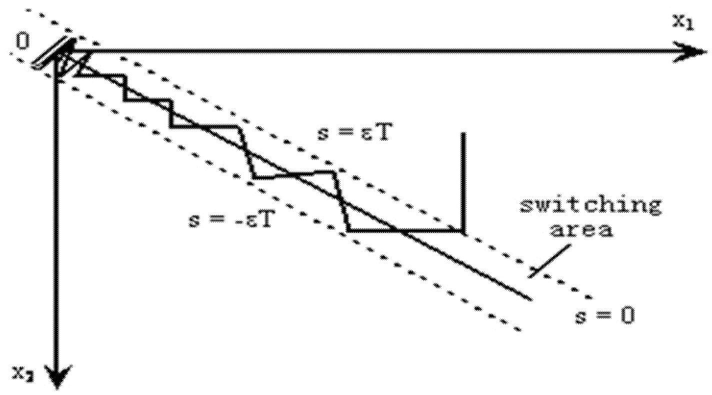

FIGURE 4. Switching area of index reaching law (II)

the advantages of them can be preserved. In details, formula (3-6) combination control law is:

$$ds(k) = \begin{cases} -\varepsilon T||x(k)||_1 sgn(s(k) - qTs(k)), & ||x(k)||_1 > k_0 \\ -\varepsilon T||x(k)||_1 sgn(s(k)), & ||x(k)||_1 \leq k_0 \end{cases} \tag{3-7}$$

where $||x(k)||_1 = \sum_{i=1}^{n} |x_i|$, k_0 must be properly chosen, if it is too big, the advantages of the variable reaching law may be hidden; if it is too small, a strong passing through shaking will occur. The selection of the turning point of the two control laws depends on actual conditions.

4. **Simulation Research and Experiment Confirmation** [5,6]**.** On our lab-platform, the voltage is 2.81V when the rotor is at its lowest position while 3.21V the highest position. By data-sample card PC-7483, we can sample the values of the voltage output by the gap detector from the very beginning to the dynamic balance state and under interference circumstances. Figure 5 show the charts of them drew by MATLAB. In order to show the advantages of the variable structure control in advance, two algorithms are used for compare: the variable structure control based on the combination reaching law and PID.

5. **Conclusions.** The discrete time variable structure controller (VSC) based on combination reaching law is designed on the basis of the mathematical model of AMB. The results of simulation and experiment show that the over-regulations of the variable structure control is lower than the PID control judging from the dynamic response when starting and under interferences, and the stable state's reaching time is also shorter, which shows

a better robust character of the variable structure controller. When the magnetic bearing system is running at the stable state, VSC can work as well as PID controller, whereas when the work condition is getting sophisticated, the performance of variable structure controller will be much better than the PID controller. Thus the method has a wide industrial application prospect.

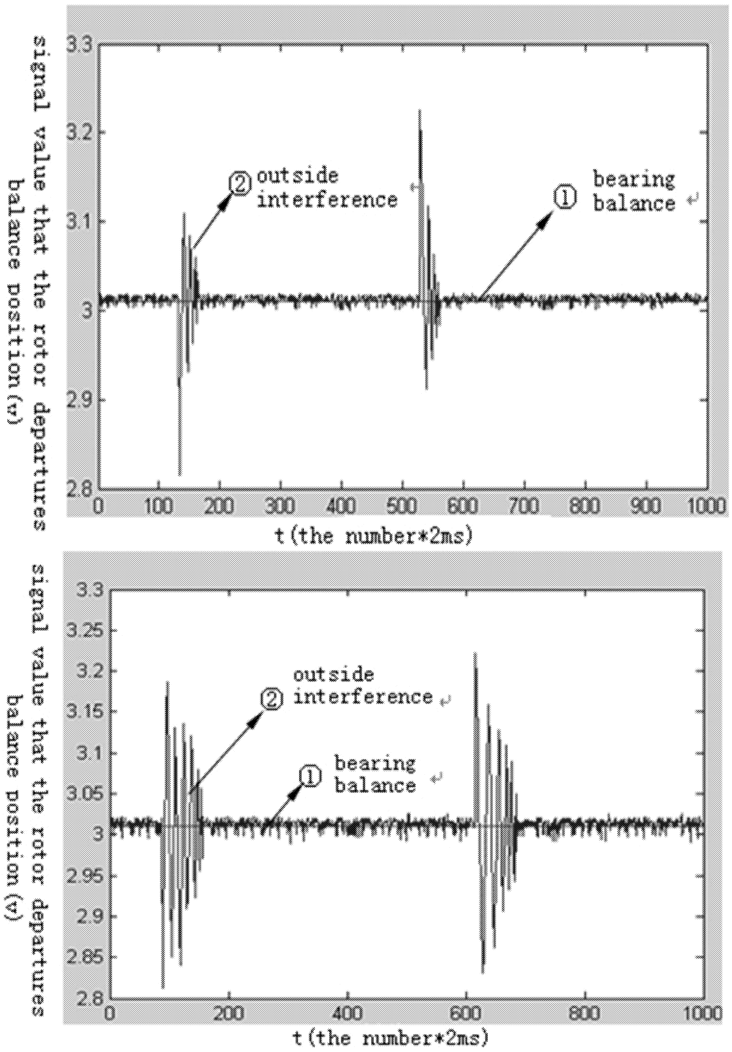

FIGURE 5. The response of the magnetic bearing system under interferences

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