EXPERIMENTAL STUDY ON SECURE COMMUNICATION OF DIFFERENT SCROLL CHAOTIC SYSTEMS WITH IDENTICAL STRUCTURE

HONG CHEN, QUN DING, LINA DING AND XUEYANG DONG

Electronic Engineer College
Heilongjiang University
Heilongjiang Harbin 150080, P. R. China
ding-qun@263.net

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ABSTRACT. Synchronization and secure communication of different scroll attractor chaotic systems are studied experimentally. Based on 5-scroll chaotic circuits, the synchronization of different scroll chaotic circuits, signal encryption and decryption are implemented by using a single variable unidirectional coupling and multi-variable unidirectional-coupled control. All measured figures are recorded by digital phosphor oscillograph TDS3032B based on the physical electronic circuits.

Keywords: Multi-scroll chaotic attractor, Secure communication, Synchronization, Unidirectional coupling

1. Introduction. The rapid development of science and technology provides convenient conditions for deciphering the encryption system, which causes secure communication a growing threat. So, it is urgent to find new and effective methods for realizing secure communication. In the early 1990s, Pecora and Carroll experimentally realized the synchronization of chaotic systems based on electronic circuits, which lead a new way to secure communication. Scientists have studied synchronization and communication of chaotic circuits and have made a lot of achievements [1-5]. In order to make the communication systems more secure, more complicate chaotic circuits were studied. Chua’s circuit is a typical chaotic circuit with its simple structure, rich chaotic actions. When its nonlinear resistance is improved, it is easy to get more complicate multi-scroll chaotic circuit [10-12]. At present, the synchronization of multi-scroll chaotic circuits is achieved by computer simulation and physical experiments [11,12]. However, all the references above mainly studied the synchronization and the secure communication of two chaotic systems of exactly same structures with the same scroll attractors. But for different surroundings of two places and chaotic sensitivity to the initial values, the same chaotic states (scroll number) will not be available. Whether can two chaotic systems with the same structure achieve synchronization and secure communication when they produce different scroll numbers? We studied and did experiments based on Chua’s 5-scroll chaos circuits. In the experiments, we choose single variable and multi-variable unidirectional couple [5,12] to control the synchronization of chaotic systems.

2. The Synchronization of Chaotic Systems. It is the key that we keep synchronization between the transmitting and receiving chaotic systems in chaotic secure communication. Only when two systems keep synchronization, the signal can be effectively obtained in the receiving system. First we choose single variable to control the synchronization and communication of two systems.

Figure 1 is the experimental circuit of multi-scroll chaotic secure communication. Circuit (1) is a chaotic circuit in the transmitting system, and circuit (2) is one in the
receiving system, their structures are the same. The circuit in dotted line frame is the unidirectional coupling control-circuit and $v_{C1}$ is regarded as the control variable to synchronize two chaotic circuits. Under synchronizing, the state equations of circuit (1) and circuit (2) are:

$$
\text{System1 : } \begin{cases}
    \frac{dC_1 v_{C1}}{dt} = v_{C2} - v_{C1} - f_D(v_{C1}) \\
    \frac{dC_2 v_{C2}}{dt} = \frac{v_{C1} - v_{C2}}{R} + i_L \\
    \frac{L di_L}{dt} = -v_{C2}
\end{cases}
$$

$$
\text{System2 : } \begin{cases}
    \frac{dC_1 v'_{C1}}{dt} = \frac{v'_{C2} - v'_{C1}}{R} - f_D(v'_{C1}) + \delta(v_{C1} - v'_{C1}) \\
    \frac{dC_2 v'_{C2}}{dt} = \frac{v'_{C1} - v'_{C2}}{R} + i'_L \\
    \frac{L di'_L}{dt} = -v'_{C2}
\end{cases}
$$

where, $f_D(v_{C1})$ and $f_D(v'_{C1})$ are piecewise linear functions separately in two chaotic circuits of Figure 1, $\delta$ is the unidirectional coupling coefficient.

**Figure 1.** The experimental circuit of multi-scroll chaotic secure communication

Let $e_1 = v_{C1} - v'_{C1}$, $e_2 = v_{C2} - v'_{C2}$, $e_3 = i_L - i'_L$, then error equations can be got by (1) and (2):

$$
\begin{cases}
    \dot{e}_1 = \frac{1}{RC_1} (e_2 - e_1) - \frac{1}{C_1} f_D(v_{C1}) - \frac{1}{C_1} f_D(v'_{C1}) - \delta e_1 \\
    \dot{e}_2 = \frac{1}{RC_2} (e_1 - e_2) + \frac{1}{C_2} e_3 \\
    \dot{e}_3 = -\frac{1}{L} e_2
\end{cases}
$$

where

$$
f_D(v_{C1}) - f_D(v'_{C1}) = k(v_{C1} - v'_{C1})
$$

the $k$ is a time-variant parameter.

From (4) and (3), we have the following equation:

$$
\begin{cases}
    \dot{e}_1 = \frac{1}{RC_1} (e_2 - e_1) - \frac{1}{C_1} k e_1 - \delta e_1 \\
    \dot{e}_2 = \frac{1}{RC_2} (e_1 - e_2) + \frac{1}{C_2} e_3 \\
    \dot{e}_3 = -\frac{1}{L} e_2
\end{cases}
$$
As (5) is time-variant equation, we analyze the stabilization of the error system (5) by Lyapunov stabilization theory. We choose a Lyapunov function as the following:

$$E = \frac{\alpha}{2} e_1^2 + \frac{\beta}{2} e_2^2 + \frac{\gamma}{2} e_3^2$$  \hspace{1cm} (6)

The $\alpha$, $\beta$ and $\gamma$ are constants.

The derivative of Eq. (6) is:

$$\dot{E} = \alpha e_1 \dot{e}_1 + \beta e_2 \dot{e}_2 + \gamma e_3 \dot{e}_3$$  \hspace{1cm} (7)

From (5) and (7), we have the following equation:

$$\dot{E} = \left[ \left( \frac{\alpha}{2RC_1} \right)^2 + \left( \frac{\beta}{2RC_2} \right)^2 - \alpha \left( \frac{1}{RC_1} + \frac{k}{C_1} + \delta \right) \right] e_1^2 + \left( 2 - \frac{\beta}{RC_2} \right) e_2^2$$

$$+ \left( \frac{\beta}{C_2} - \frac{\gamma}{L} \right) e_2 e_3 - \left( \frac{\alpha}{2RC_1} \right)^2 e_1 - \left( \frac{\beta}{2RC_2} \right)^2 e_2$$  \hspace{1cm} (8)

So only satisfying following inequality

$$\begin{align*}
\left\{ \begin{array}{l}
\left( \frac{\alpha}{2RC_1} \right)^2 + \left( \frac{\beta}{2RC_2} \right)^2 - \alpha \left( \frac{1}{RC_1} + \frac{k}{C_1} + \delta \right) \leq 0 \\
2 - \frac{\beta}{RC_2} \leq 0 \\
\frac{\beta}{C_2} - \frac{\gamma}{L} = 0
\end{array} \right.
\end{align*}$$  \hspace{1cm} (9)

Then $\dot{E}$ must be a negative definite. According to Lyapunov stability, we know that error system (5) is asymptotically stable. Namely, $v_{C1}$ unidirectional coupling control can make system (1) and system (2) synchronized asymptotically.

By the same way we can prove the synchronization of 2-variable and 3-variable unidirectional coupling control.

3. The Experimental Study of Synchronization and Secure Communication.

The $v_{C1}$ in Figure 1(1) is regarded as the control variable to control $v'_{C1}$ in circuit 1(2) with unidirectional coupling, and makes the two systems synchronized. The chaotic signal $v_{C2}$ is used to encrypt signal to mixed with original signal $S(t)$ for forming cipher text and then be sent out. In the receiving system, chaotic signal $v'_{C2}$ is used as decrypt signal and will abstract the original signal $S'(t)$ from cipher text when the two systems synchronizing.

3.1. Single variable unidirectional coupling control. The $R$ and $R'$ are regulated separately in Figure 1(1) and 1(2), so, both of the circuits can generate a 5-scroll attractor, which can be seen in the upper right corner and upper left corner of Figure 2(a); the middle one is the phase diagram of $v_{C2}$ versus $v'_{C2}$ when there is no synchronization between the systems in Figure 1(1) and 1(2). From the disorderly chaotic phase diagrams, it can be concluded that though two chaotic systems are exactly same, they cannot be synchronized without any control.

When switch $K_3$ is closed, namely under synchronizing, the phase diagrams of $v_{C1} - v_{C2}$ and $v'_{C1} - v'_{C2}$ are shown in the upper right corner and upper left corner of Figure 2(b), they are two exactly same 5-scroll attractors; the synchronization phase diagram of $v_{C2} - v'_{C2}$ is shown in the middle of Figure 2(b), which is a 45 degree straight line. All of the above show that two chaotic systems can achieve synchronization under unidirectional coupling control.

The upper square wave signal in Figure 2(c) is the original signal $S(t)$ to be encrypted, after encrypting in transmitting system and being sent out. The lower waveform in Figure 2(c) is decrypted the signal $S'(t)$ in the receiving system under two systems synchronizing.
From the experiment, we can see that two chaotic systems with the same scroll are easy to synchronize and realize secure communication.

![Figure 2. Same scroll chaotic attractor systems](image)

3.1.1. The communication between 5-scroll and 3-scroll attractor chaotic systems. Figure 1(1) produces a 5-scroll attractor, which is shown in the upper left corner in Figure 3(a). And Figure 1(2) produces a 3-scroll chaotic attractor by adjusting $R_N$, which is shown in the upper right corner in Figure 3(a) (phase diagram of $v_C^1 - v_C^2$). The phase diagram of $v_C^1 - v_C^2$ without synchronization is very chaotic, as shown in Figure 3(a).

When synchronization control, Figure 1(1) and 1(2) will be synchronized to a straight line of $45^\circ$ (phase diagram of $v_C^2 - v_C^2$) which is shown in Figure 3(b). At the same time, the attractor in Figure 1(2) is changed from a 3-scroll attractor to a 5-scroll attractor, shown in the upper right corner in Figure 3(b). So, 5-scroll and 3-scroll systems can also be synchronized.

From original signal $S(t)$ and decrypted signal $S'(t)$ in Figure 3(c), it can be clearly seen that the communication can be achieved between a 5-scroll and a 3-scroll systems, but there is slightly bigger disturbance on the signal $S'(t)$.

![Figure 3. 5-scroll and 3-scroll chaotic attractor systems](image)

3.1.2. The communication between 5-scroll chaos and single scroll chaotic systems. Figure 1(1) still generates a 5-scroll attractor (shown in upper left corner in Figure 4(a)). The circuit in Figure 1(2) is adjusted to a single scroll attractor, as shown in upper right corner in Figure 4(a). The phase diagram $v_C^2 - v_C^2$ of the two systems without synchronization is shown in Figure 4(a).

When switch $K_3$ is closed, the two chaotic circuits achieve synchronization, which change the single scroll attractor in Figure 1(2) into a 5-scroll attractor, and get a $45$ degree synchronous line, shown as Figure 4(b). We can see that 5-scroll and single scroll systems can also be synchronized.

From Figure 4(c) it also can be seen the effects of synchronization communication between 5-scroll and single scroll attractor systems. But, compared with the former two cases, the interruption on this signal $S'(t)$ is the biggest, as shown in Figure 4(c).
3.2. The multi-variable unidirectional coupling control. When $v_{C1}$ and $i_L$ are used as two control variables by using unidirectional coupling to control $v'_{C1}$ and $i'_L$ separately (the same as [15], Figure 9), the synchronizing figure of $v_{C2} - v'_{C2}$, original signal $S(t)$ and decrypted signal $S'(t)$ are shown in Figure 5. When $v_{C1}$, $v_{C2}$ and $i_L$ are used as control variables, and control $v'_{C1}$, $v'_{C2}$ and $i'_L$ with unidirectional coupling, the synchronizing figure of $v_{C2} - v'_{C2}$, original signal $S(t)$ and decrypted signal $S'(t)$ are shown in Figure 6. In Figures 4, 5 and 6, they are all the synchronization and secure communication between 5-scroll and 1-scroll chaotic attractor systems by the unidirectional coupling control. and the more the control variables, the better the effects of secure communication.

4. Conclusions. The various synchronization and communication effects are concluded and listed in Table 1. From No.3, 4 and 5 in Table 1, we can see that they have bigger difference in the number of scroll attractor which produced by two systems separately. But if we choose proper control variables, the actual demand effect of synchronization communication can be achieved definitely. Although the more different the number of the scrolls, the worse the decryption effects, the more control variables are used, the better the synchronization communication between two systems will be, as shown in Table 1.
It is found experimentally that two systems of generating different scrolls under different conditions can achieve the synchronization communication by a proper synchronizing method.

Table 1. The various synchronization and communication effects

<table>
<thead>
<tr>
<th>No.</th>
<th>Variables</th>
<th>The scrolls of two chaotic systems</th>
<th>S/N</th>
<th>The results of synchronization and secure communication</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1-variable</td>
<td>5-scroll and 5-scroll</td>
<td>1000</td>
<td>the better result of synchronization and communication, see Figure 2</td>
</tr>
<tr>
<td>2</td>
<td>1-variable</td>
<td>5-scroll and 3-scroll</td>
<td>500</td>
<td>can synchronize and communicate, but worse than 1, see Figure 3</td>
</tr>
<tr>
<td>3</td>
<td>1-variable</td>
<td>5-scroll and 1-scroll</td>
<td>294</td>
<td>can synchronize and communicate, but worse than 1 and 2, see Figure 4</td>
</tr>
<tr>
<td>4</td>
<td>2-variables</td>
<td>5-scroll and 1-scroll</td>
<td>1000</td>
<td>2-variables better than 1's, see Figure 5</td>
</tr>
<tr>
<td>5</td>
<td>3-variables</td>
<td>5-scroll and 1-scroll</td>
<td>1667</td>
<td>the best result of synchronization and secure communication, see Figure 6</td>
</tr>
</tbody>
</table>

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