TUNING DECENTRALIZED PID CONTROLLERS FOR PERFORMANCE AND ROBUST STABILITY

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ABSTRACT. In this paper a frequency domain method for tuning decentralized PID controller for nominal performance and robust stability is proposed. An appropriate formulation of the frequency domain nominal performance and robust stability conditions enables us to transform the design problem into an optimization task to be possibly dealt with using genetic algorithms (GA). Robust decentralized PID controller design for the glass tube manufacturing plant using the proposed approach is included.

Keywords: Genetic algorithm, Nominal performance, PID controller, Robust stability

1. Introduction. Industrial plants are complex systems typical by multiple inputs and multiple outputs (MIMO systems) and interactions. MIMO systems can be controlled by either 1) multivariable or centralized controllers (if there are strong interactions within the system) or 2) a set of SISO decentralized controllers. Despite certain performance deterioration due to the restricted controller structure the latter approach is preferred in practice because of hardware and operation simplicity and reliability improvement. With the come up of the robust frequency domain approaches in the 80’s, the robust approach to the decentralized controller design has become very popular and many practice-oriented approaches were developed along with the computationally useful tools used to assess the closed-loop performance under decentralized controllers.

In general, performance can be assessed from two perspectives [8]: according to the “global perspective” general characteristics and measures are observed yielding more relevant results about the closed-loop dynamic than the “response perspective” which evaluates particular responses. When designing decentralized control for performance, the performance objectives can be of two basic types depending on the specific application: a) achieving a required performance in the different subsystems (using either independent or dependent design methods); b) achieving a desired performance of the overall system using dependent design.

In this paper a frequency domain robust decentralized PI controller design for nominal performance and robust stability is presented. The proposed procedure is of dependent design-type, i.e. all controllers are designed simultaneously. To guarantee nominal performance (NP) of the full system the condition for the weighted sensitivity function is applied; to guarantee robust stability in terms of additive uncertainty the $M$-$\Delta$ structure based stability condition is applied. Both conditions suitably modified have been included in the fitness function of the developed genetic algorithm to obtain the resulting robust decentralized PID controller.

The paper is organized in Preliminaries and Problem Statement where main approaches to assess robust stability and nominal performance in decentralized systems are surveyed and the problem addressed is formulated; in Main Results the applied genetic algorithm
2. Preliminaries and Problem Statement. Consider an uncertain plant described by a transfer function matrix \( G(s) \in \mathbb{R}^{m \times m} \), and a diagonal controller \( R(s) \in \mathbb{R}^{m \times m} \) in the standard feedback configuration (Figure 1).

![Figure 1. Standard feedback](image)

Let the family of possible realizations of the uncertain plant be given as a set of \( N \) transfer function matrices corresponding to \( N \) different operating points, hence

\[
G_k(s) = \{G_{ij}^k(s)\}_{m \times m}, \quad k = 1, 2, \ldots, N
\]

where \( y_i^k(s) \) is the \( i \)-th output and \( u_j^k(s) \) is the \( j \)-th input in the \( k \)-th operating point.

2.1. Robust stability conditions. Unstructured uncertainty associated with the system model (1) can be described using any of the three uncertainty forms [7]; due to the least conservative results the additive perturbation has been chosen (Figure 2) generating the related family of plants \( \Pi_a \):

\[
\Pi_a : \quad G(s) = G_N(s) + \ell_a(s)\Delta(s)
\]

\[
\ell_a(s) = \max_k \{ \sigma_{\text{max}}[G^k(s) - G_N(s)] \}
\]

where \( G_N(s) \) is the nominal model, \( \sigma_M(\cdot) \) is the maximum singular value of \( (\cdot) \) and \( \Delta(s) \) is the perturbation such that \( \sigma_M(\Delta) \leq 1 \). The nominal model can be obtained as the mean value parameter model from models in different working points [7] or affine-type nominal model when the uncertain plant is modelled using the affine-type additive uncertainty [5].

To examine closed-loop stability robustness, the feedback loop with the uncertain system is transformed into the \( M-\Delta \) structure (Figure 2); under the assumption that both the nominal system \( M(s) \) and the perturbation \( \Delta(s) \) are stable, the \( M-\Delta \) system is stable for all perturbations satisfying \( \sigma_{\text{max}}[\Delta(j\omega)] \leq 1 \) if and only if [7]

\[
\sigma_{\text{max}}[M(j\omega)] < 1, \quad \forall \omega
\]

If additive uncertainty is used then

\[
M(j\omega) = M_a(j\omega)\ell_a(\omega)
\]

where \( \ell_a(\omega) \) is given by (2) and

\[
M_a(j\omega) = -[I + R(s)G_N(j\omega)]^{-1}R(j\omega)
\]

and additive uncertainty condition (3) becomes

\[
\sigma_{\text{max}}(M_a(j\omega)) < \frac{1}{\ell_a(\omega)}
\]
2.2. Frequency domain performance specifications. Before deciding whether to use a fully interacting or a decentralized controller it is necessary to check whether it is possible to achieve the desired performance objectives with the latter one. Several criteria and tools have been proposed to assess applicability of the decentralized controller. Plant-model based tools for control structure selection for single loop controllers include the relative gain array (RGA), Niederlinski index, AV index [3], μ interaction measure; the performance relative gain array (PRGA) allows understanding the effect of directions under DC [1]. Closed-loop performance measures e.g. the closed-loop disturbance gain (CLDG) and the decentralized relative gain (dRG) are used to estimate the effect of interactions under finite bandwidth decentralized control.

A commonly used closed-loop performance measure for both SISO and MIMO systems is the sensitivity function

\[ S(s) = (I + G(s)R(s))^{-1} \]  

Basic performance specification are expressed in terms of the weighted sensitivity function

\[ \sigma_{\max}(W_P S) < 1 \text{ or } \sigma_{\max}(S(j\omega)) < \frac{1}{|w_p(j\omega)|}, \quad \forall \omega \]

where \( W_P = \text{diag}\{w_{pi}\}_{i=1,...,m} \) and \( w_p(s) \) is a scalar performance weight that specifies the maximum allowed magnitude of \( \sigma_{\max}(S(j\omega)) \) at each frequency. Typical specification in terms of \( S \) include minimum bandwidth \( \omega_B^* \) (defined as the frequency at which \( \sigma_{\max}(S(j\omega)) \) crosses the value 0.707 from below), maximum tracking error, steady state control error, shaping \( \sigma_{\max}(S(j\omega)) \) over selected frequency ranges, maximum peak max \( \max_{\omega} \{\sigma_{\max}[S(j\omega)]\} \leq M_S \) that prevents noise amplification at high frequencies and serves as a robustness margin; typically \( M_S = 2 \). The corresponding weight \( w_p(s) \) is e.g.

\[ w_p(s) = \frac{s \omega^*_B + \omega_B^*}{s + \omega^*_B A} \]

where \( |w_p(j\omega)| \), (the upper bound on |S|) is equal to \( A \) at low frequencies (typically \( A \approx 0 \)) and equal to \( M \geq 1 \) at high frequencies; the asymptote crosses 1 at the frequency \( \omega_B^* \) which is approximately the bandwidth requirement.

Search for decentralized PID controller parameters guaranteeing simultaneous fulfillment of robust stability and nominal performance conditions can be formulated as optimization task and solved using genetic algorithms (GA). A simple block scheme of GA is shown in Figure 3 [6].

2.3. Problem formulation. Consider an uncertain MIMO system with \( m \) subsystems (1). A robust decentralized controller

\[ R(s) = \text{diag}\{R_i(s)\}_{i=1,...,m}, \quad \det R(s) \neq 0, \quad \forall s \]
$P_0$-initial population, $U$-group of unchanged strings, $P_k$- population in the $k$-th generation, $G$-group of strings for genetic operations (parents), $B$-group of best strings, $G'$-group of $G$ after genetic operations (children).

Figure 3. Block scheme of GA

is to be designed with $R_i(s)$ being transfer function of the $i$-th local controller. The designed controller has to guarantee stability within the plant operating range described by the perturbed plant model (2), and a required nominal closed-loop performance.

3. Main Results. To be able to use the GA to design a robust decentralized controller, the design problem has to be recast to optimization task with the cost function including three main components accounting for nominal stability, nominal performance and robust stability. Such an approach allows to combine both the “global” and the “response” perspectives to performance evaluation in the fitness function.

The proposed design procedure is of dependent design-type, i.e. local PID controllers are designed simultaneously. The design is carried out for the full nominal model obtained using any available method; in this paper the mean value parameter model calculated from models in individual working points has been used. After obtaining the nominal model, the additive perturbation is obtained according to (2).

3.1. Guaranteeing robust stability (RS). To guarantee robust stability in terms of additive uncertainty, the $M$-$\Delta$ based stability condition (6) is applied. Modifying (5) as follows

$$Ma = [I + RG_N]^{-1}R = [R(R^{-1} + G_N)]^{-1}R = (R^{-1} + G_N)^{-1}$$

and using properties of singular values allows to reformulate (6) as follows

$$\sigma_{\min}[(R^{-1} + G_N)] > \ell_a(\omega)$$

Using (12) allows avoiding the matrix inversion in (6) because $R(s)$ is a diagonal matrix.

3.2. Guaranteeing nominal performance (NP). To guarantee nominal performance of the full system, condition (8) is applied for the weighted nominal sensitivity

$$\sigma_{\max}(SN(j\omega)) < \frac{1}{|w_p(j\omega)|}, \text{ where } SN = [I + G_NR]^{-1}$$

or equivalently

$$\sigma_{\min}(I + G_NR) > |w_p(j\omega)|$$

An appropriate weight $w_P(s)$ (9) is to be chosen and specified in terms of $A$, $\omega^*_B$ and $M$ with respect to the plant dynamics.
3.3. **Guaranteeing nominal stability (NS).** The NS requirement is implicitly included in the requirement for nominal performance.

3.4. **The proposed design procedure.** includes specifications of the frequency range, space of feasible PID controller parameters, \(w_P(s)\) (9) and \(\ell_\alpha(\omega)\) (2).

In the GA setup, a chromosome is a set of \(m\) randomly generated triplets of local PID controller parameters ordered respectively to the order of individual loops. The fitness \(F\) includes evaluation (and if necessary penalization) of the fulfillment of the robust stability and the nominal performance conditions (12) and (14), respectively. The procedure stops after fulfillment of appropriate termination conditions. The convergence rate depends on the search space size and dimension, GA structure and on the used genetic operations.

4. **Example.** Consider a two-input two-output model of the glass tube manufacturing plant [4] described in three working points. The corresponding mean value parameter model is

\[
G_N = \begin{bmatrix}
-245.7s + 982.7 & 2.2s - 4.227 \\
\frac{40}{s + 4.567} & \frac{32}{s^2 + 9.567s + 22.83}
\end{bmatrix}
\]

The objective is to design local PI controller parameters with the transfer function in the parallel form, guaranteeing setpoint step tracking over the whole operation range of the plant specified by the three operating points. Calculation of the Niederlinski index \(N = 1.0662\) proves that the nominal system is stabilizable using the decentralized controller. Frequency dependent RGA proved a correct pairing up to \(\omega \approx 2s^{-1}\). Nominal performance weight \(w_p(s)\) was specified with: \(A = 10^{-4}, \omega_p^* = 1.4s^{-1}, M = 2\). The GA was run with 100 generations per 30 populations. Parameters of individual PI controllers have been chosen from \(r_0i \in <0.02, 0.1>; r_{-1}i \in <0.02, 0.1>, i = 1, 2\). Resulting local controllers are:

\[
R_1(s) = 0.0429 + \frac{0.0859}{s}, \quad R_2(s) = 0.1027 + \frac{0.4829}{s}
\]

Closed loop step responses under the designed controller are in Figure 4; verification of the robust stability condition is in Figure 5.

5. **Conclusion.** In this paper a frequency domain method for tuning decentralized PID controller for nominal performance and robust stability has been proposed. An appropriate formulation of the frequency domain nominal performance and robust stability
conditions enables to transform the design problem into an optimization task to be possibly dealt with using genetic algorithms.

The proposed frequency domain GA-based robust decentralized PID controller design technique is a depend design-type technique guaranteeing a desired performance of the overall full nominal system. Nominal performance criteria entering into the fitness are “general perspective” ones, hence they do not depend on particular configuration of input signals (e.g. reference steps). The success of the procedure strongly depends on a priori information about the controlled plant, choice of the performance weight parameters as well as on a thorough selection of feasible ranges of individual controller parameters (the larger is the search space the more generations are needed in the GA and hence the larger is the computing time).

The proposed GA-based approach can further be improved using more subtle performance measures referred to e.g. in [1] and developed also for independent-type design.

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REFERENCES