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INFORMATION AND ENERGETICS OF QUANTUM FLAGELLA MOTOR

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Abstract. *We proposed a sample of stochastic motor, which moved based on principle of stochastic resonance. First we introduced a Lagrangian, having dissipative terms caused by water molecules. Using Eular-Lagrange method, we get the motion of equations for motA-membranes complex of flagella motor. Proton–pumps attached in cell -membranes make proton electro static potential gradients according written as Nernst Equation. It is said that their potential gradients accelerate and flow in protons through a base of flagella motor. Under the process of the diffusion into cell body caused by protons gradient, many protons collide with motA complex imbedded into cell-membranes. We can consider eight numbers of the motA complex as a kind of a closed ring chain model connected with springs, because of the motA complex combined with a elastic cell-membranes. Each motA complex connected with elastic membranes give rise to stochastic resonance, what is call, lattice vibration. That vibration corresponds to a correlative motion in classical physics, and its phenomena called coherent state in quantum mechanics. This quantum coherent state generated by external force (collision of protons) lowers potential energy curves of spring. From that coherent motion, the ground state is not in the lowest energy state, but in the upper stated from the point of the reduced potential energy curves. We see same phenomena in elementary particle physics as a phase transition of vacuum energy, and the energy of old ground state is higher than that of new one. The old ground state naturally releases its excess energy in itself. The electrostatic potential generated by proton pump and the stochastic resonance (coherent state) caused by the collision of protons are always used for maintaining those shift of ground state, and the reduction of potential energy.*
Keywords: Flagella motor, Stochastic resonance, Proton pump, Coherent state, Correlative motion, MotA, Lower potential curve

1. **Introduction.** We have been proposing mechanism of actin myosin system and prototype quantum flagella motor system using thermal noise, which called stochastic resonance [1-4]. Generally speaking, it is said that many motor proteins utilize the energies of hydrolysis of ATP or those of gradient of proton for their motions. The actin-myosin systems of muscles need ATP molecules for the contraction of muscles. On the other hand, flagella motor commonly spends protons flux instead of ATP molecules [5-8]. However, those two systems adopt thermal noise and stochastic resonance [3,4]. Many models of flagella motor based on theories of supersonic motors have been proposed [9]. Those models are based on theories of classical rigid body and a kind of supersonic models. However, those supersonic models do not clearly refer to a role of proton flux, and they have nothing to

do with quantum theory in spite of phenomena of nano-systems (Figure 1).

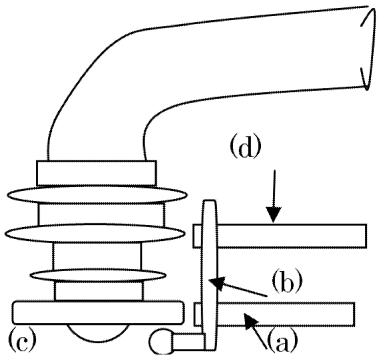

FIGURE 1. Flagella motor

(a) cell-membrane, (b) motA-fliG, (c) fliF (M ring), (d) glycan layer
Many motA penetrate both membranes, (a) and (d). The eight numbers of motA are imbedded into those elastic membranes. So, we can regard the membranes as a kind of springs connecting among those motA.

In this paper, we show the proton flux that caused stochastic resonance between motA and springs. The vibrating motA gives rise to lattice vibration that is a kind of coherent motion. The lattice motion reduces the ground state energy, and it shifts downward to lower energy level. Thus, the energetic level of the previous ground state is higher than that of the new one. We show the stochastic resonance always emerges from phenomena of common impacts, and stochastic resonance called in micro-region corresponds to coherent state in quantum mechanics.

The motA have a kind of ionic channels, and many protons collide with their walls of motA when protons go through the motA ionic channels.

The collision of protons with motA being induced by electrostatic potential gradients between inner and outer cell-membranes works out the stochastic resonance (coherent state) and the reduction of the energetic level. The difference between the old energetic level and the new one, what is called, a kind of phase transition of vacuum in elementary particle physics, practices to maintain the motion of motA. The eights motA imbedded in cell-membranes make circular motions or elliptic ones. The motA contacting with a M-rings (fliF) deliver the motion of motA to the M-ring like as supersonic motors.

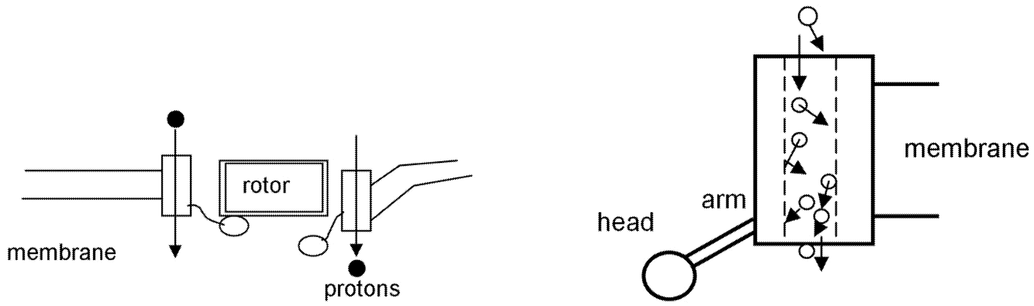

FIGURE 2. FliG-motA complex

The motA have ionic channels. The protons go through those ionic channels by electrostatic membrane potentials induced by energetic proton pumps of flagella. The FliG-motA-complex vibrate with together when protons go through ionic channel of motA

1) Protons collision causes vibrations
2) Ring chain's vibrations are lattice motion
3) Making cooperative motions which is called stochastic resonance

2. **Motion of Flagella Motor.** It is interesting to analyze a rotational mechanism for flagella motor as well as a motion of actin-myosin system of muscles. We would like to

propose new flagella motor model based on stochastic resonance and potential energetic shift. First of all, we show the Lagrangian of motA containing an external force and dissipative systems (many water molecules).

Flagella motor generates the rotational power with proton flux caused by the diffuse process of hydrogen ions. The eight numbers of motA are imbedded into elastic cell-membranes and they penetrate stiff glycan layer. So we names the construction being composed of three elements (glycan layer, motA, cell-membrane) as motA-membranes complex. The motA have a kind of ionic channel, and each motA is connected with the elastic cell-membrane and stiff glycan layer. We consider that motA-membranes complex as a closed ring chain model with springs connecting each motA.

The flux of proton in water flows into intra cellar flagella body through the eight numbers of motA's ionic channels. Then its flux interacts with motA-membranes complexes of flagella motor, and each proton collides with motA's channels, and each motA irregularly vibrates. The spectrums of vibrating modes contain all of frequency, what is call, white noise, since each collision between a proton and motA-membranes complex is caused at random. Then Lagrangian of flagella motor is described as

$$\begin{aligned} L = \sum_{n=1}^{N} \tfrac{m}{2}\dot{\vec{q}}_n^2 - \sum_{n=1}^{N} \tfrac{k}{2}(\vec{q}_{n+1} - \vec{q}_n)^2 + \sum_{n=1}^{N} \tfrac{M}{2}\dot{\vec{Q}}_n^2 - \sum_{n=1}^{N} \tfrac{K}{2}(\vec{Q}_{n+1} - \vec{Q}_n)^2 \\ + \sum_{n=1}^{N} \vec{f}_n \cdot \vec{q}_n - \sum_{n=1}^{N} \alpha_n(\vec{Q}_n \cdot \vec{q}_n)^2 \end{aligned} \tag{1}$$

We think that the model of the motA–membranes complex of the flagella motor is a kind of the closed spring-chain model. Imbedded N numbers of motA into the membrane are regarded as N's mass points, i.e. in our case, $N = 8$. The first and second term of Lagrangian mean the kinetic and potential energy of each motA-membranes complex. We adopt springs' potential energies of harmonic potentials. And the third and fourth terms are corresponding to the heat reservoirs (dissipative parts), and they represent the coordinates of many clusters of water molecules. All of energies in flagella motors are absorbed in those water molecules that play a role of huge dissipative terms. The fifth term represents the mechanism of the generating powers with collisions between each proton and each motA, and the last term represents the dissipative process of between the motA-membranes complexes and water molecules, and their energies flow from the motA-membranes complexe to much of water molecules through this trem. The real flagella motor has eight motA ($N = 8$) imbedded into two kinds of layers, whose structures are regarded as the closed spring-chain model. Using Eular-Lagrange method, we get the motion of equations for motA-membranes complexes,

$$m\ddot{q}_{n,i} = -k\left(q_{n+1,i} - q_{n,i} - 2q_{n,i}\right) + f_{n,i} - 2\alpha_{n,i}Q_{n,i}(\vec{Q}_{n,i} \cdot \vec{q}_{n,i}) \tag{2}$$

The $i = 1, 2, 3$ means x, y, z components of coordinate. And dissipative parts, which are water molecules, are described as

$$m\ddot{Q}_{n,i} = -K\left(Q_{n+1,i} - Q_{n,i} - 2Q_{n,i}\right) - 2\alpha_{n,i}Q_{n,i}(\vec{Q}_{n,i} \cdot \vec{q}_{n,i}) \tag{3}$$

Total Hamiltonian with both dissipative part and power generating one becomes

$$\begin{aligned} H = \sum_{n=1}^{N} \tfrac{m}{2}\vec{p}_n^2 + \sum_{n=1}^{N} \tfrac{k}{2}\left(\vec{q}_{n+1} - \vec{q}_n\right)^2 + \sum_{n=1}^{N} \tfrac{M}{2}\vec{P}_n^2 + \sum_{n=1}^{N} \tfrac{K}{2}\left(\vec{Q}_{n+1} - Q_n\right)^2 \\ - \sum_{n=1}^{N} \vec{f}_n \cdot \vec{q}_n + \sum_{n=1}^{N} \alpha_n\left(\vec{Q}_n - q_n\right)^2 \end{aligned} \tag{4}$$

We introduce normal difference coordinate into Equation (2), and this frame has an orthogonality and completeness, and then it is expressed as

$$q_{n,i} = \sum\nolimits_K a_{K,i}(t) \cdot u_{n,i}^K(\vec{x}) \tag{5}$$

where the index K counts the members of the basis. A natural and convenient choice is the use of harmonic function

$$u_{n,i}^{K} = \frac{1}{\sqrt{N}} \exp(ik_i a_i n) \tag{6}$$

which makes Equation (5) a discrete Fourier decomposition. The index k_i has the dimension of an inverse length and corresponds to the wave number of the plane wave Equation (5). Thus k_i must satisfy

$$k_i = \frac{2\pi}{Na_i} l, \quad -\frac{N}{2} < l < \frac{N}{2}, \quad l = 1, 2, 3, \ldots \tag{7}$$

In order to analyze the vibrating motion of motA, we pay attention to harmonic terms at Equation (2). The Equation (2) has a harmonic oscillator solution with a frequency

$$\omega_{k,i} = 2\sqrt{\frac{k_i}{m}} \left| \sin \frac{k_i a_i}{2} \right| \tag{8}$$

This equation is called the dispersion relation of chain oscillations. Thus, we get the explicit solution with using normal coordinate systems.

$$q_{n,i} = \sqrt{\frac{1}{N}} \sum\nolimits_{k,i} [b_{k,i} \exp\{-i(\omega_{n,i}t - k_i a_i n)\} + b_{k,i}^{*} \exp\{i(\omega_{n,i}t - k_i a_i n)\}] \tag{9}$$

The second term is the complex conjugate to the first one so that the coordinate is indeed real. The knowledge of the normal coordinates can be used to construct solutions to the initial value problem for the coordinate $q_{n,i}(t)$. If it were not for the third parts and dissipative part of Equation (2), then that equation has an approximate solution with initial conditions $q_n(0) q_n'(0)$,

$$q_{n,i} = \frac{1}{N} \sum_{mk,i} \left[q_m(0) \cos\{k_i a_i (n-m) - \omega_{k,i} t\} - \frac{1}{\omega_{k,i}} \dot{q}_{n,i}(0) \sin k_i a_i (n-m)\} - \omega_{k,i} t \right] \tag{10}$$

The above solution expresses the motion of motA

$$\vec{q}_n(t) = (q_{n,x}, q_{n,y}, q_{n,z}) \tag{11}$$

If the spring constant $k_x = k_y = k_z$, then the motion of motA imbedded into elastic membranes draws the circle orbital or sphere one like as the lattice motion of piezoelectric device. If those motA contact with fliF (M-ring) which is a rotor of flagella motor, then they can transmit their circular motions to the M-ring.

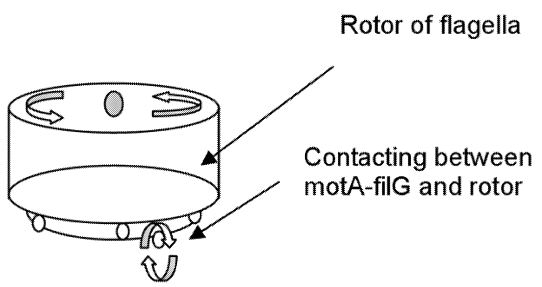

FIGURE 3. Motion of flagella rotors

Eight motA are imbedded into elastic membranes named as complex of cell membrane and gylcan laye.When motA -FliG complexes are looked upon as the closed spring-chain model, heads of motA-FliG complexes are made to circulate by the lattice motion of the closed spring-chain. If the heads of motA-FliG-membrane complexes contact with M-ring of flagella motor, the circular is transmitted to the rotor of flagella.

Thus, the M-ring practice to rotate same as supersonic motors using raveling wave. And the motions of motA give rise to corelative motion without an external force and dissipative terms.

3. **External Force and Coherent State.** We showed that actin-myosin system utilizes the thermal noise and stochastic resonance in our previous papers [1-4]. In those papers, myosin head is approximately described by nonlinear equations as shown in following styles

$$m\ddot{x} = -\frac{\partial V}{\partial x} - \gamma_0 \dot{x} + F(t) \tag{12}$$

where potential V is non linear interaction between actin and myosin head, and myosin head, $F(t)$ means collision of water molecules ($\delta(t)$, delta function)), γ_0 is dissipative coefficient(viscous). So, we showed that the acti-myosin system propels to one direction with stochastic resonance by using the approximate solution.

$$X \approx c + \frac{A_1 t}{2\mu} + \frac{A_1 \sin\gamma t}{2\mu\gamma} - \frac{A_1(\mu\cos\gamma t + \gamma\sin\gamma t)}{2\mu(\mu^2+\gamma^2)} \cdots \tag{13}$$

We notice that the second term corresponds to translational motion, and third vibration part is stochastic resonance effect between myosin head and thermal noise. We especially emphasize that an impact $\delta(x)$, collisions of water molecules and non-linear potentials (non linear force), and springs cause the stochastic resonance and translation motions of systems. And we show that impact $\delta(t)$ has a coherent state in quantum mechanics.

If the harmonic oscillator's part and potential V do not depend on an explicit time development, then we can get Schrödinger equation taken no account H_0 as far as we pay attention to much short while ε. This equation is given in

$$i\hbar\frac{d}{dt}\left|\varphi(t)\right\rangle = \left(H_0 + f(t)V\right)\left|\varphi(t)\right\rangle \tag{14}$$

Performing integration, the Equation (14) gives us time development description of state vector

$$\left|\varphi(t)\right\rangle = \exp\left[-\frac{i}{\hbar}\left(\int_{t_0}^{t} U(t')\right)dt'\right]\left|\varphi(t_0)\right\rangle = \exp\left[-\frac{i}{\hbar}\left(\int_{t_0}^{t} f(t')\right)dt'V\right]\left|\varphi(t_0)\right\rangle \tag{15}$$

If an impact (potential of collision) between water molecules (a single proton) and a myosin head (one motA) has the following relationship with delta function

$$U = -\delta(t)\cdot f_n x_n \tag{16}$$

Then, we can find out the coherent expression with state vector according to Schrödinger equation and a number description (creation operator a^+) for harmonic oscillator. The state vector is written as

$$\left|\varphi(\varepsilon)\right\rangle = \exp\left(\frac{i}{\hbar}f_{n,}x_n\right)\left|\varphi(0)\right\rangle = \exp\left(\frac{-1}{4m\hbar\omega}f_n^2\right)\exp\left(\sqrt{\frac{1}{2m\hbar\omega}}f_n a_n^+\right)\left|\varphi(0)\right\rangle \tag{17}$$

So, we can conclude that common impacts cause a coherent state (stochastic resonance) in quantum mechanics. The coherent state of quantum mechanics is equal to stochastic resonance in classical mechanics.

Our Equation (2) have harmonic oscillation term (springs), non-linear term (an external force, collision term), and dissipative term. Thus, our equation of flagella motor, the Equation (2) can give rise to the coherent state (stochastic resonance) in quantum mechanics.

In next step, we would like to get the number description for harmonic terms of motA. From Equation (2), we can express in matrix form as

$$\vec{q}(t) = kA\vec{q} \tag{18}$$

and where vector q is an N-dimensional vector. Since only direct neighbors in the chain interact with each other the N-N coupling matrix $\boldsymbol{A}$ is tridiagonal. The harmonic oscillators parts of Hamiltonian of Equation (2) reads

$$H = \sum\nolimits_{n=1}^{N} \frac{m}{2}\vec{p}_n^2 + \sum\nolimits_{n=1}^{N} \frac{k}{2}\left(\vec{q}_{n+1} - \vec{q}_n\right)^2 \tag{19}$$

where matrix $\boldsymbol{A} = \begin{bmatrix} -2 & 1 & 0 & 0 \\ 1 & -2 & \ddots & 0 \\ 0 & \ddots & \ddots & 1 \\ 0 & 0 & 1 & -2 \end{bmatrix}$.

To make the transition from classical to quantum mechanics, one replaces the position and momentum coordinate of point masses by linear operators, and introduces commutation relations for number operators.

$$q_{n,i\cdot} = \sum\nolimits_k \sqrt{\frac{\hbar}{2m\omega_{n,i}}}(c_{k,i}u_{n,i}^{k} + c_{n,i}^{+}u_{n,i}^{k+}), \quad p_{n,i\cdot} = -i\sum\nolimits_k \sqrt{\frac{\hbar}{2m\omega_{n,i}}}(c_{k,i}u_{n,i}^{k} - c_{n,i}^{+}u_{n,i}^{k+}) \tag{20}$$

We substitute Equation (20) into Equation (19), and we get Hamiltonian for number orators.

$$H = \sum\nolimits_{\hbar\omega_{n,i}} \left(c_{n,i}^{+}c_{n,i} + \frac{1}{2}\right), \quad [c_{k,i}, c_{n,j}^{+}] = \delta_{n,k}\delta_{i,j}, \quad [c_{k,i}^{+}, c_{n,j}^{+}] = [c_{k,i}, c_{n,j}] = 0 \tag{21}$$

The quantized vibrations of system, caused by correlation motions, are called phonons. Now the chain is subjected to the influence of an externally applied force. This force f_n of Equation (1) varies in space but it is assumed to be constant in time and the potential V is to the spatial coordinate q_n. The electrostatic potential energy in Equation (1) is given by

$$\begin{aligned} V = -\sum\nolimits_n f_{n,i}q_{n,i} &= -\sum\nolimits_n F_i \sum\nolimits_{k,i} \sqrt{\tfrac{\hbar}{2m\omega_{k,i}}}\left(c_{k,i}u_n^{k,i} + c_{k,i}^{+}u_n^{+k,i}\right) \\ &\equiv \sum\nolimits_{k,i}\left(F_{k,i}c_{k,i} + F_{k,i}^{+}c_{k,i}^{+}\right) \end{aligned} \tag{22}$$

From Equation (18), we get the complete Hamiltonian reads

$$H = \sum\nolimits_n \hbar\omega_{n,i}\left[\left(c_{n,i}^{+}c_{n,i} + \frac{1}{2}\right) - \left(F_{k,i}c_{k,i} + F_{k,i}^{+}c_{k,i}^{+}\right)\right] \tag{23}$$

The interaction operator is linear in the creation and annihilation operators. We introduce a shift operators by suitable c numbers $\alpha_{k,i}$, and define new creation and annihilation operators through

$$d_{k,i} = c_{k,i} - \alpha_{k,i}, \quad d_{k,i}^{+} = c_{k,i}^{+} - \alpha_{k.i}^{+} \tag{24}$$

The Hamiltonian (23) becomes in the new operators,

$$H = \sum\nolimits_n \hbar\omega_{n,i\cdot}\left[\left(d_{n,i}^{+}d_{n,i} + \frac{1}{2}\right) - |\alpha_{k,i}|^2\right] \tag{25}$$

The only influence is simultaneous lowering of the eigen energies by the value $|\alpha_{k,i}|^2$. The drop in energy correspond to the work expended by the external force in order to bring the system to the new configuration. The old ground state $|0\rangle$ changes into the new ground state $|0, \alpha_{k,i}\rangle$, which is leads to

$$|n, \alpha_{k,i}\rangle = \prod\nolimits_k \frac{1}{\sqrt{n_{k,i}}}(d_{k,i}^{+})^{nk,i}\,|0, \alpha_{k,i}\rangle \tag{26}$$

This notation show that the eigen states of interacting motA depend on the force parameter $\alpha_{k,i,}$ and interacting and non-interacting ground states do not agree $|0, \alpha_{k,i}\rangle$ is not equal to $|0\rangle$ (Figure 2).

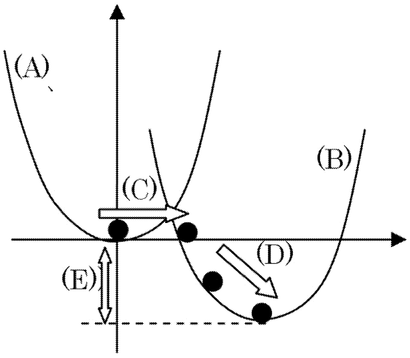

FIGURE 4. Shift of potential cirve

(A) old potential curve. (B) new potential curve. (C)-(E) The influence of an external force lowers the potential energy curve of a harmonic oscillators but does not change its shape. The black ball is in the old ground state at first (C). Then the energy curve (A) shifts to lower state that is named as the new ground state (B). And the black ball in the old ground state does not have the lowest energy level at the new ground state. So, the black ball roll down into the lowest energy level (the new ground state) (D). The lowest ground state maintain spending the electrostatic potential of the membranes (cell-membrane and glycan layer) being generated by proton pumps of the flagella.

The ground state of motA according to Equation (25) is given by the coherent superposition of infinitely multi-phonon states.

$$|0, \alpha_{k,i}\rangle = \exp(-\frac{1}{2}|\alpha_{k,i}|^2 \sum\nolimits_{nk,i} \frac{1}{\sqrt{n_{k,i}}} \alpha_{k,i}^{nk,i} |n_{k,i}\rangle \tag{27}$$

The coherent state of motA is introduced as the eigen states of the annihilation operator, and the expectation of the position operator (20) coherent state,

This is a finite value that represents the final state displacement reached under the influence of external force.

$$\bar{q}_{n,i} = \frac{1}{\sqrt{N}} \sum\nolimits_k \sqrt{\frac{\hbar}{2m\omega_{k,i}}} \cdot 2Re\left[\alpha_{k,i} \exp i(kan - \omega_k t)\right] \tag{28}$$

The source of the external force f_n comes from the electrostatic membranes potential between inner and outer membranes. We adopt Nernst relation,

$$\vec{f}_n = e\vec{E} = -e\frac{V}{L} = -e\frac{RT}{LF} \ln \frac{P_{in}[H^+]}{P_{out}[H^+]} \approx -59\Delta pH \tag{29}$$

where L is thickness of membrane. The f_n are the forces of motA generated by the collisions of protons which are accelerated by voltage potential difference. The driving energy of single motA-fliG-membranes complex is written down as

$$V_i = -\sum\nolimits_n f_{n.i} q_{n,i} = -\sum\nolimits_n \left(eq_{n,e} \frac{RT}{LF} \ln \frac{P_{in}[H^+]}{P_{out}[H^+]} \right) \approx -59 \sum\nolimits_n q_{n,i} \Delta pH \tag{30}$$

Total potential reads

$$U = \sum\nolimits_i V_i \delta(t - t_i) \approx -59 \sum\nolimits_n \sum\nolimits_i q_{n,i} \delta(t - t_i) \Delta pH \tag{31}$$

Note that the physical principle of flagella motor does not belong to classical mechanics, but to quantum mechanics. When we can consider applying quantum physics to flagella motor, we can find out the shift of energetic state and coherent state. In order to maintain the shift of energetic state, we recognize to utilize the proton pumps and the electrostatic membranes potential being defined by Nernst relation.

4. **Summary.** We proposed a model of stochastic motor, which moved by using principle of coherent state (stochastic resonance). Many protons accelerated by voltage potential difference collide with motA. The motA imbedded membranes vibrate proper frequencies. And the flagella motors begin to rotate when the motA take circular motions like as supersonic motor. Those mechanisms correspond to reduce the energetic levels (shift of energy curves) with using corelative motions.

1) The proton pumps make the electrostatic potential on the two membranes. (cell-membrane and gylcan layer)
2) Many protons inflows into flagella body through the basis of flagella. Then, the protons in the waters collide with motA imbedded in the two elastic membranes.
3) So the motA-fliG begin to select and to vibrate the proper frequencies, which is stochastic resonance in classical mechanics.
4) We showed the stochastic resonance is equal to coherent state in quantum mechanics with the way of quantum theory of the impact.
5) The coherent states reduce the old energetic level and its ground state to the new energetic level and the new ground state. We think that the energy curve shift and a phase transition are looked upon as a kind of the coherent.
6) An energy of the difference between the new energetic level and the old energetic level are released, and the energy is spent for circulating motions of motA-fliG. The motA-fliG contact with fliF (M ring), so the fliF start to rotate like as supersonic motors of traveling wave types.
7) The proton pumps continue to supply the energies for the sake of maintaining the reduction of the energy curve.

REFERENCES

[1] H. Matsuura and Nakano, Stochastic model of actin-myosin system, new stochastic resonance inclined rods model having double springs, *J. of BMFFSA*, vol.8, no.1, pp.1-13, 2002.
[2] H. Matsuura and M. Nakano, Theoretical approach to actin-myosin system as an example of complex system, *J. of BMSFA*, vol.3, no.1, pp.47-52, 1997.
[3] H. Matsuura, N. Noda, K. Koide, T. Nemoto and M. Nakano, Stochastic dynamics in biological system and information, *International Journal of Innovative Computing, Information and Control*, vol.4, no.2, pp.233-248, 2008.
[4] M. Nakano, H. Matsuura, N. Noda and K. Koide, Stochastic inclined rods model for nano-size protein motors in muscle, *International Journal of Innovative computing, Information and Control*, vol.4, no.2, pp.275-282, 2008.
[5] W. Kohn, *Highlight of Condensed Matter Theory*, North-Holland, 1985.
[6] C. Jones and S. Aizawa, The bacterial flagellum and flagllar motor, adv., *Microbiol. Physiol.*, vol.32, pp.109-172, 1991.
[7] D. Blair and H. Berg, Restoration of torque in defective flagellar motors, *Science*, vol.242, pp.1678-1681, 1988.
[8] Y. Sowa, S. Fukuoka and M. Honnma, Single molecule technique on the rotation and function in the supra-molecular motor flagella, *Protein, Nucleic Acid and Enzyme*, vol.52, no.4, pp.309-316, 2007
[9] T. Atsumi, An ultrasonic motor model for bacterial fllagellar motors, *J. Theor. Bil.*, vol.213, pp.31-51, 2001.